Optimal Multiple Vehicle Motion Planning

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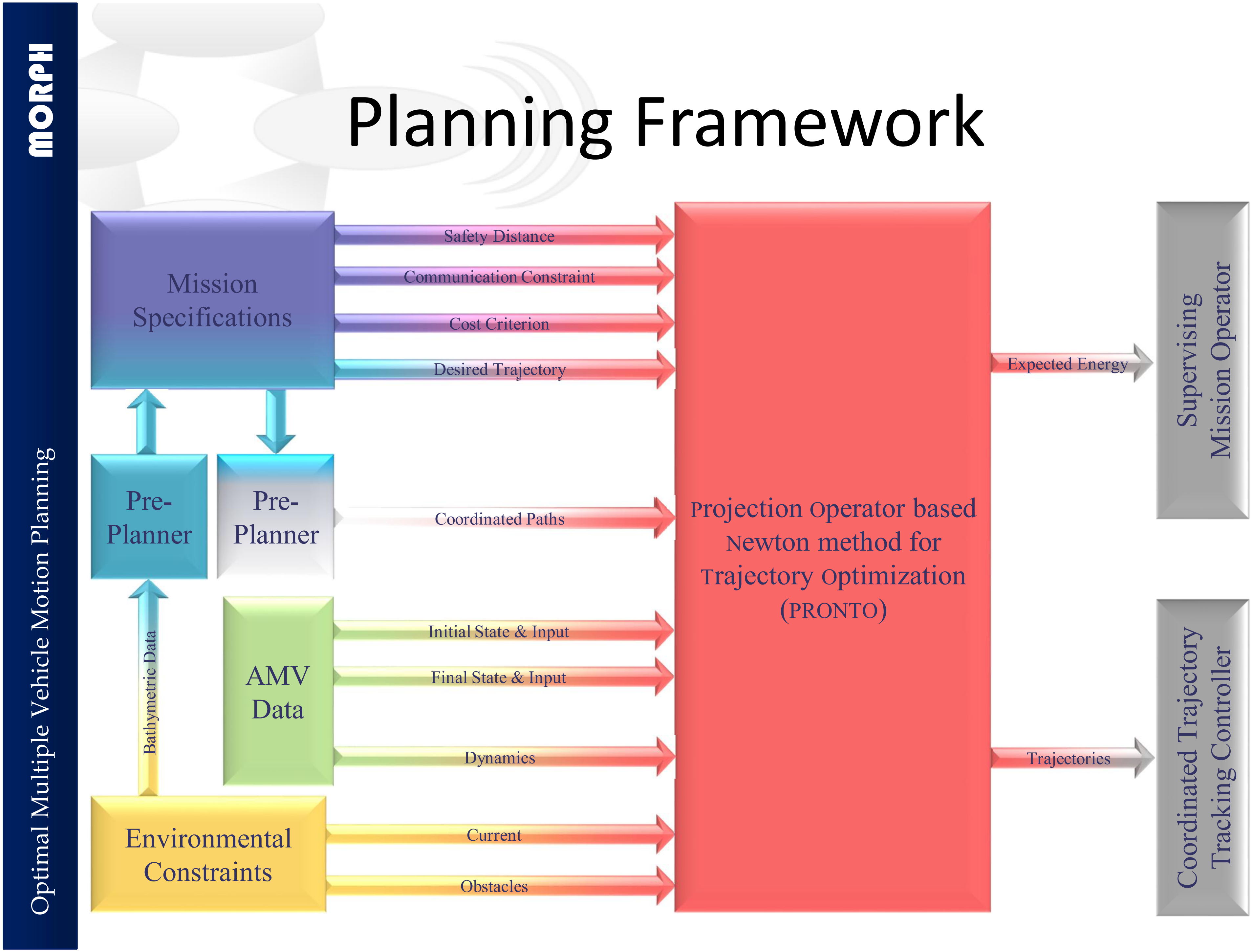
Problem Setting

- Vision: Groups of autonomous vehicles freely roaming the oceans
- Objectives: Acquire data on unprecedented scale; detect and monitor episodic events; inspect critical infrastructure on permanent basis
- Requirement: Planning a mission that can be properly executed with minimal energy expenditure
- Challenges: Simultaneous planning for several vehicles; possibly heterogeneous team configuration; inter-vehicle and obstacle collision avoidance; spatial team configuration; …

Main Features

- Explicitly incorporated nonlinear vessel dynamics
- Four-quadrant thruster model for energy consumption and propulsion calculation
- Using a descent method for solving constrained continuous-time optimal control problems
- Pre-planners for collision avoidance and terrain-based trajectory generation

Planning Framework



Vehicle model, optimization constraints.

THE OPTIMIZATION PROBLEM

Dynamic Model

- Kinematics

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}$$

- Dynamics

$$(M_{\mathrm{rb}} + M_{\mathrm{a}})\dot{\nu} + (C_{\mathrm{rb}}(\nu) - C_{\mathrm{a}}(\nu))\nu + (D + D_{\mathrm{n}}(\nu))\nu = \tau$$

where $\nu = \begin{bmatrix} u \\ v \\ r \end{bmatrix}$ and $\tau = \begin{bmatrix} T_{\mathrm{ps}} + T_{\mathrm{sb}} \\ 0 \\ l(T_{\mathrm{ps}} - T_{\mathrm{sb}}) \end{bmatrix}$

- Inputs c_{ps} and c_{sb}, the portside and star board propeller's rate of change of the rotational velocities

Thruster Model

- Due to design: negligible propeller-hull interaction

- Four-quadrant propeller model

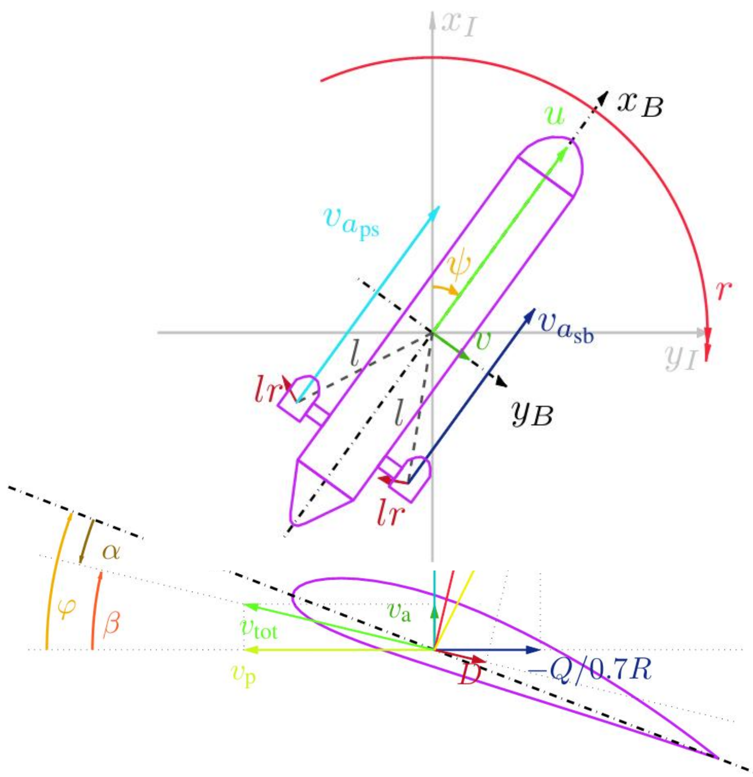
$$T = \frac{1}{2} c_{\mathrm{T}}(\beta)\left(v_{\mathrm{a}}^2 + v_{\mathrm{p}}^2\right)\pi R^2$$

$$Q = \frac{1}{2} c_{\mathrm{Q}}(\beta)\left(v_{\mathrm{a}}^2 + v_{\mathrm{p}}^2\right)\pi R^2 d$$

where

$$\mathrm{v}_{\mathrm{a}_{\mathrm{ps}}} = -lr + u$$

$$\mathrm{v}_{\mathrm{a}_{\mathrm{sb}}} = -lr + u$$



- Dynamics

$$\dot{n}_{\mathrm{ps}} = c_{\mathrm{ps}}$$

$$\dot{n}_{\mathrm{sb}} = c_{\mathrm{sb}}$$

D/C Motor Model

- Standard DC motor equations

$$L_a \frac{dI}{dt} + R_a I = V - K_e \omega$$

$$J_m \dot{\omega} + b\omega = K_t I_a - Q_{hyd}$$

- Assume sufficiently fast dynamics $\rightarrow$ steady state equation for voltage

$$V = R_a I + K_e \omega$$

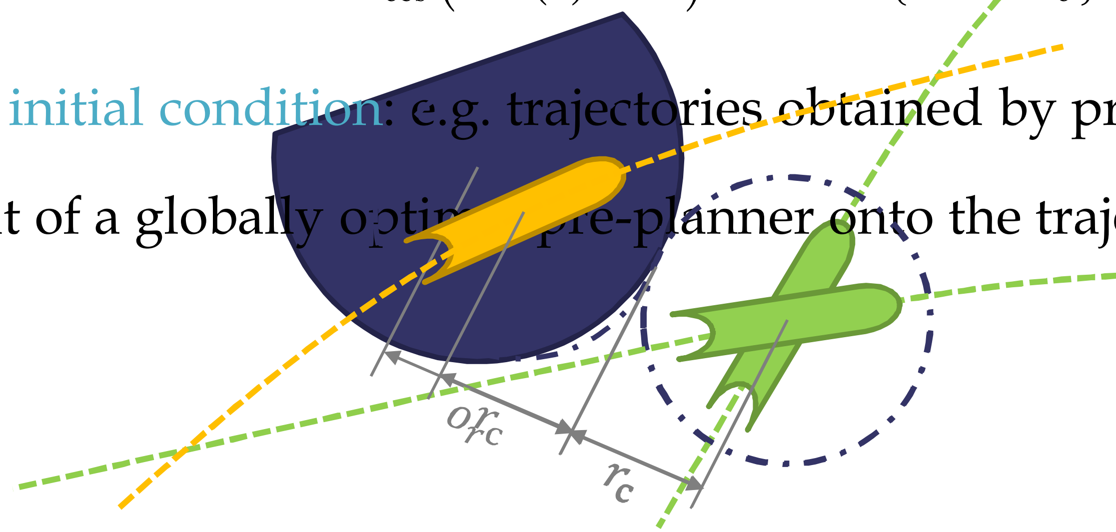
- Quasi-static model for electrical current

$$I_a = \frac{1}{K_t}\left(b\omega + Q_{hyd}\right)$$

where K_t and b are obtained by nonlinear least squares fit to measurement data

The Complete Problem

- cost functional $\min \int_0^T \sum_{i=1}^{N_v} \left(l_{\text{pow}}\left(\mathbf{x}^{[i]}(\tau), \mathbf{u}^{[i]}(\tau)\right) + l_{\text{des}}\left(\mathbf{x}^{[i]}(\tau), \mathbf{u}^{[i]}(\tau), \tau\right)\right) d\tau + m\left(\mathbf{x}(T)\right)$
- subject to constraints
  - dynamics $\dot{\mathbf{x}}^{[i]} = f\left(\mathbf{x}^{[i]}, \mathbf{u}^{[i]}, t\right)$, $\mathbf{x}^{[i]}(0) = \mathbf{x}_0^{[i]}$, $\mathbf{x}^{[i]}(T) = \mathbf{x}_{\text{f}}^{[i]}$
  - collision avoidance $c_{\text{col}}\left(\mathbf{x}^{[i]}(t), \mathbf{x}^{[j]}(t)\right) \geq 0$, $i, j \in \{1, \ldots, N_{\text{v}}\}$, $i \neq j$
  - obstacle avoidance $c_{\text{obs}}\left(\mathbf{x}^{[i]}(t), \mathbf{o}^{[k]}\right) \geq 0$, $k \in \{1, \ldots, N_{\text{o}}\}$
- with initial condition: e.g. trajectories obtained by projecting the result of a globally optimal pre-planner onto the trajectory manifold



Trajectory optimization using the projection operator approach.

PRONTO – AN INTRODUCTION

Minimization of Trajectory Functionals

Consider the problem of minimizing a functional

$$h\left(x(\cdot),u(\cdot)\right)=\int_0^T l\left(x(\tau),u(\tau),\tau\right)d\tau+m\left(x(T)\right)$$

over the set $\mathcal{T}$ of bounded trajectories of the nonlinear system

$$\dot{x}(t)=f\left(x(t),u(t),t\right),\qquad x(0)=x_0$$

Here, $\mathrm{x}(t)\in\mathbb{R}^n$ and $u(t)\in\mathbb{R}^m$.

We write this constrained problem as

$$\min_{\xi\in\mathcal{T}} h(\xi)$$

where

- $\xi=\left(\alpha(\cdot),\mu(\cdot)\right)$ is a bounded curve with $\alpha(\cdot)$ continuous
- $\xi\in\mathcal{T}$ means $\dot{\alpha}(t)=f\left(\alpha(t),u(t)\right)$ and $\alpha(0)=x_0$

Projection Operator Approach

Key idea: a trajectory tracking controller may be used to minimize the effects of system instabilities, providing a numerically effective, redundant trajectory parametrization.

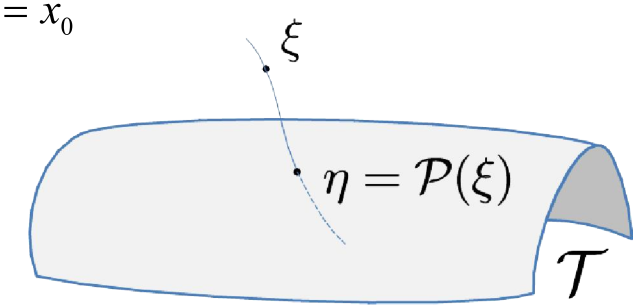
- Let $\xi(t) = \big(\alpha(t), \mu(t)\big)$, $t \geq 0$, be a bounded curve.
- Let $\eta(t) = \big(x(t), u(t)\big)$, $t \geq 0$, be the trajectory of f determined by the nonlinear feedback system

$$\dot{x} = f(x,u), \qquad x(0) = x_0$$
$$u = \mu(t) + K(t)\big[\alpha(t) - x\big]$$

- The map

$$\mathcal{P} : \xi = \big(\alpha(\cdot), \mu(\cdot)\big) \mapsto \eta = \big(x(\cdot), u(\cdot)\big)$$

  is a continuous, nonlinear projection operator.



[Figure by courtesy of A. Saccon]

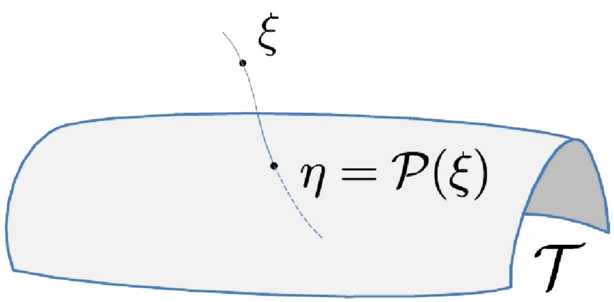
- For each $\xi \in \operatorname{dom} \mathcal{P}$, the curve $\eta = \mathcal{P}(\xi)$ is a trajectory. (The trajectory contains both state and control curves.)

Projection Operator Properties

Suppose that f is C^r and that K is bounded and exponentially stabilizes $\xi_0 \in \mathcal{T}$. Then

- $\mathcal{P}$ is well defined on an L_∞ neighborhood of ξ_0
- $\mathcal{P}$ is C^r (Fréchet differentiable wrt. L_∞ norm)

- $\mathcal{P}(\xi) \in \mathcal{T}$ for all $\xi \in \operatorname{dom} \mathcal{P}$
- $\xi \in \mathcal{T}$ if and only if $\xi = \mathcal{P}(\xi)$
- $\mathcal{P} = \mathcal{P} \circ \mathcal{P}$ (projection)

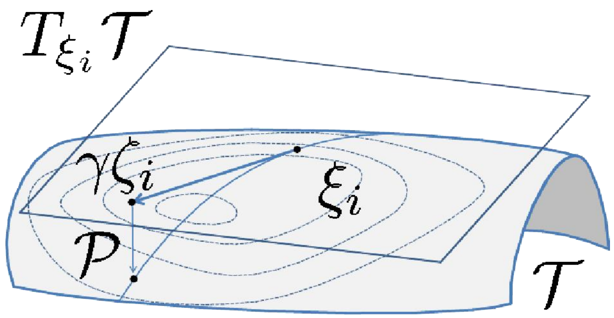


[Figure by courtesy of A. Saccon]

- On the finite interval $[0,T]$, choose $K(\cdot)$ to obtain stability-like properties so that the modulus of continuity of $\mathcal{P}$ is relatively small.
- On the infinite horizon, instabilities must be stabilized in order to obtain a projection operator; consider $\dot{x} = x + u$.

Trajectory Manifold

Theorem: $\mathcal{T}$ is a Banach manifold. Every $\eta \in \mathcal{T}$ near $\xi \in \mathcal{T}$ can be uniquely represented as $\eta = \mathcal{P}(\xi + \zeta)$, with $\zeta \in T_\zeta \mathcal{T}$.



[Figure by courtesy of A. Saccon]

Key: the continuous linear projection operator $D\mathcal{P}(\xi)$ provides the required subspace splitting.

Note: $\zeta \in T_\xi \mathcal{T}$ if and only if $\zeta = D\mathcal{P}(\xi) \cdot \zeta$

The Representation Theorem provides a (local) linear parametrization of (nonlinear) trajectories.

Equivalent Optimization Problems

Composing the cost functional with the projection operator to obtain the unconstrained trajectory functional

$$g(\xi) = h(\mathcal{P}(\xi))$$

[Remember that $h(x(\cdot),u(\cdot)) = \int_0^T l(x(\tau),u(\tau),\tau)\,d\tau + m(x(T))$.]

for $\xi \in \mathcal{U} \subset \operatorname{dom} \mathcal{P}$, we see that

$$\underbrace{\min_{\xi \in \mathcal{T}} h(\xi)}_{\text{constrained}} \text{ and } \underbrace{\min_{\xi \in \mathcal{U}} g(\xi)}_{\text{unconstrained}}$$

are equivalent in the sense that

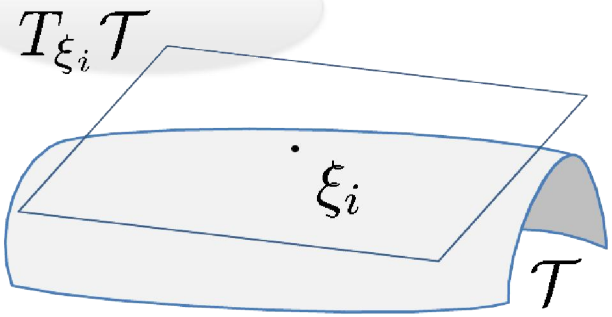
- if $\xi^* \in \mathcal{T} \cap \mathcal{U}$ is a constrained local minimum of h, then it is an uncostrained local minimum of g; and
- if $\xi^+ \in \mathcal{U}$ is an unconstrained local minimum of g in $\mathcal{U}$, then $\xi^* = \mathcal{P}(\xi^+)$ is a constrained local minimum of h.

PRojection Operator based Newton method for Trajectory Optimization (PRONTO)

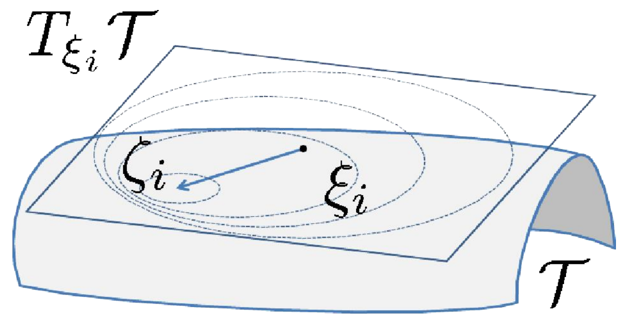
This allows the development of Newton descent methods for the optimization of h over $\mathcal{T}$, as every $\eta \in \mathcal{T}$ near $\xi \in \mathcal{T}$ can be uniquely represented as $\eta = \mathcal{P}(\xi+\zeta)$, where $\zeta \in T_{\xi}\mathcal{T}$ (i.e. $\zeta(t)$ is a tangent vector).

At each iteration, we construct and minimize a second order aproximation of g around the current trajectory ξ_i; the minimization is restricted to the tangent space.

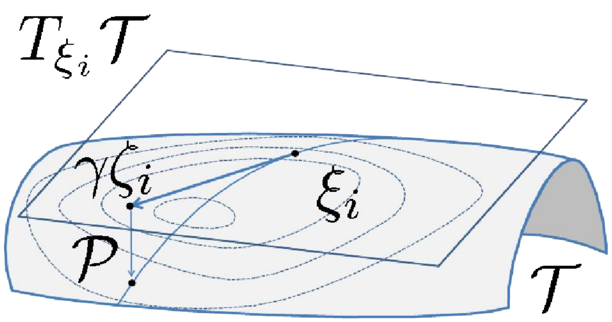
PRojection Operator based Newton method for Trajectory Optimization (PRONTO)



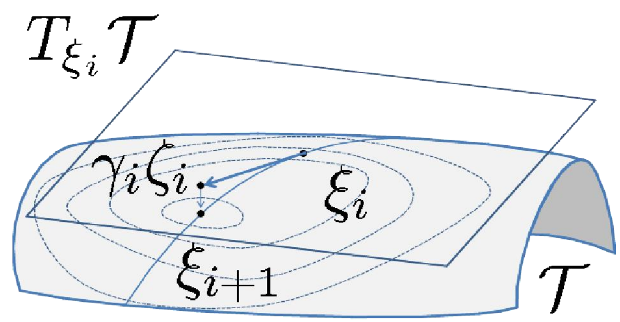
Trajectory Manifold



Descent Direction



Line Search



Update

[Figures by courtesy of A. Saccon]

PRojection Operator based Newton method for Trajectory Optimization (PRONTO)

given initial trajectory $\xi_0 \in \mathcal{T}$

for $i = 0, 1, 2, \ldots$

design feedback $K(\cdot)$, defining $\mathcal{P}$ about ξ

search direction $\zeta_i = \arg \min_{\zeta \in T_{\xi_i} \mathcal{T}} Dh(\xi_i) \cdot \zeta + \frac{1}{2} D^2 g(\xi_i) \cdot (\zeta, \zeta)$

line search $\gamma_i = \arg \min_{\gamma \in [0,1)} h\big(\mathcal{P}(\xi_i + \gamma \zeta_i)\big)$

update $\xi_{i+1} = P(\xi_i + \gamma_i \zeta_i)$

end

PRojection Operator based Newton method for Trajectory Optimization (PRONTO)

This direct method generates a descending trajectory sequence in Banach space, with quadratic convergence to second-order sufficient minimizers.

When $D^2 g(\xi_i)$ is not positive definite on $T_{\xi_i}\mathcal{T}$, one may obtain a quasi-Newton descent direction by solving

$$\zeta_i = \arg\min_{\zeta \in T_{\xi_i}\mathcal{T}} Dh(\xi_i)\cdot\zeta + \frac{1}{2} q(\xi_i)\cdot(\zeta,\zeta)$$

where $q(\xi_i)$ is positive definite on $T_{\xi_i}\mathcal{T}$ (e.g. an approximation to $D^2 g(\xi_i)$).

The β_δ barrier and the hockey stick.

BARRIER FUNCTIONAL

The β_δ barrier functional

- Key difficulty with standard log barrier methods: infeasibility of ξ is not tolerated
- Therefore: not possible to evaluate the cost functional unless ξ is a feasible curve
- Even if ξ is feasible, we cannot be sure that $\mathcal{P}(\xi)$ is

The β_δ barrier functional

- Solution: define, for $0 < \delta \leq 1$, the approximate log barrier function

$\beta_\delta : (-\infty, \infty) \to (0, \infty)$ as

$$\beta_\delta(z) = \begin{cases} -\log z & z > \delta \\ \frac{1}{2}\left[\left(\frac{z-2\delta}{\delta}\right)^2 - 1\right] - \log\delta & z \leq \delta \end{cases}$$

- Use the approximate barrier functional

$$b_\delta(\xi) = \int_0^T \sum_j \beta_\delta\left(c_j\left(\alpha(\tau), \mu(\tau), \tau\right)\right) d\tau$$

to add the constraints

$$\min_{\xi \in \mathcal{T}} h(\xi) + \epsilon b_\delta(\xi)$$

- Note: $h(\cdot) + \epsilon b_\delta(\cdot)$ can be evaluated on any curve ξ in $\tilde{X}$.

The β_δ barrier functional

- $\beta_\delta(z)$ retains many of the important properties of the log barrier function $z \mapsto -\log z$ while expanding the domain of finite values from $(0,\infty)$ to $(-\infty,\infty)$

- Now: use the projection operator based Newton method to optimize the functional

$$g_{\epsilon,\delta} = h(\mathcal{P}(\xi)) + \epsilon \beta_\delta(\mathcal{P}(\xi))$$

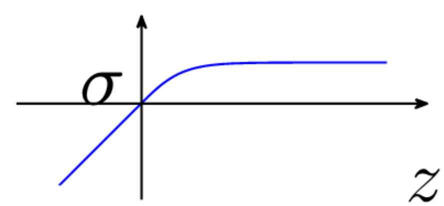
as part of a continuation method

The "Hockey Stick" Extension

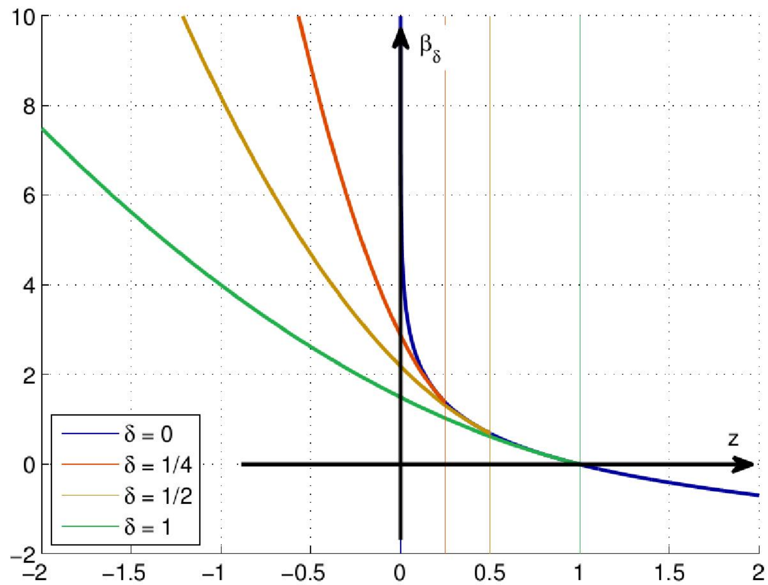
- Problem: $\beta_\delta(z)$ assumes (unbounded) negative values for $z > 1$, putting an undesired reward into collision avoidance constraint (In collision avoidance, being far away is not better than being merely feasible!)

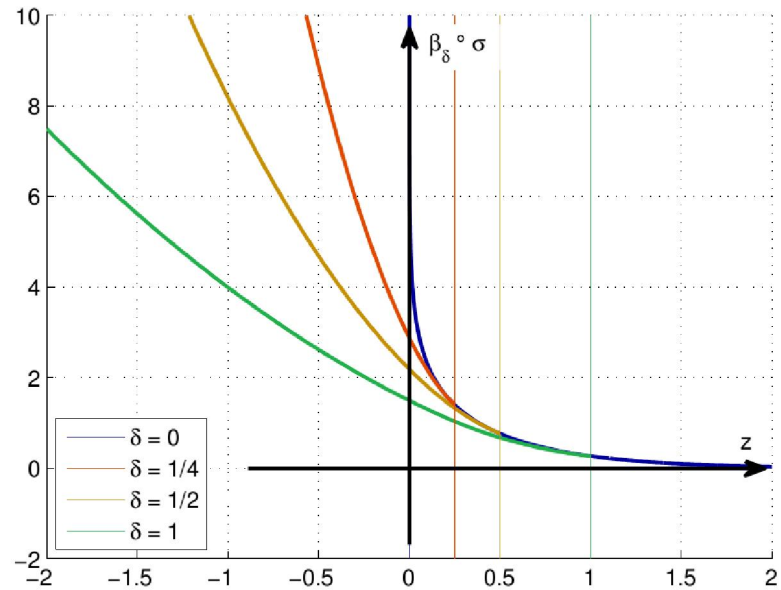
- Solution: extend β_δ barrier functional by forming a composition with the C^2 "hockey stick"

$$\sigma(z) = \begin{cases} \tanh(z) & z \geq 0 \\ z & \text{otherwise} \end{cases}$$



β_δ barrier w/hockey stick





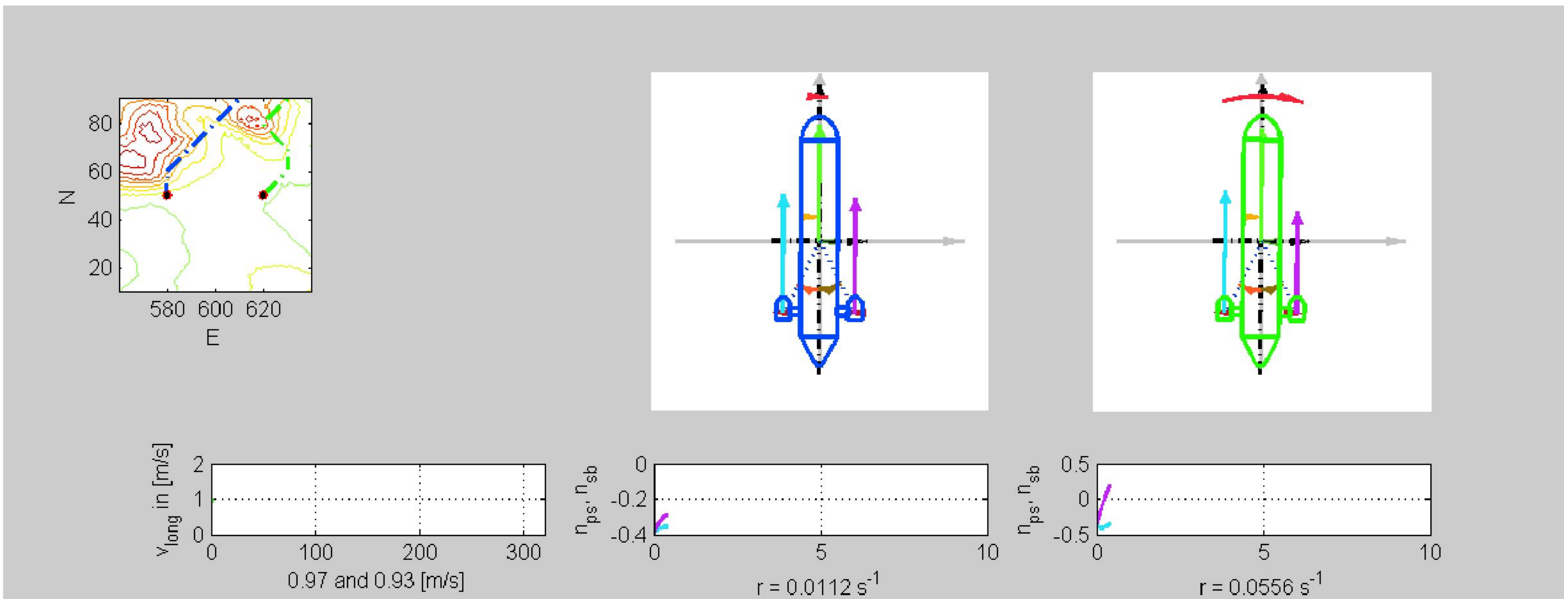
MORPH

Optimal Multiple Vehicle Motion Planning

Results and research outlook.

CONCLUSION

Result of Example Problem



Summary

- Minimum energy motion planning algorithm with
  - explicitly incorporated vehicle dynamics;
  - coordination space pre-planner;
  - first-order thruster dynamics; and
  - terrain-based pre-planning
- Future work is aimed towards
  - trajectory generation and optimization for rigid formations of vehicles;
  - bi-cubic interpolation of terrain and integration into PRONTO itself;
  - include communication between vehicles to allow for per-vehicle trade-off in terrain information to the benefit of the whole formation; and
  - extend coordination space to static obstacles and "circumvention decisions"

Thank you for your attention!

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